Some formulas you may need:

$$EV = \mu = \sum xp(X = x) \qquad Var = \left[\sum x^2 p(X = x)\right] - \mu^2 \qquad \sigma = \sqrt{\left[\sum x^2 p(X = x)\right] - \mu^2}$$

1. (2, 3, 2, 1) In this problem we are going to analyze the "field" bet in craps (where you roll a pair of dice). If you are playing craps and make the field bet,

You will win twice your bet if you roll a total of 2
You will win the amount that you bet if you roll a total of 3, 4, 9, 10, or 11
You will win three times your bet if you roll a total of 12
You will lose your bet if you roll anything else

Suppose you bet $100 on the field bet. Let X denote the amount of money you win when playing this game once.

a) Find the probability distribution for X.

x	$P(X=x)$
$200	$\frac{1}{36}$
$100	$\frac{14}{36}$
$300	$\frac{1}{36}$
-$100	$\frac{20}{36}$

b) Find the expected value, variance and standard deviation of X.

Expected Value

$$\mu = \sum x \, P(X=x) = (200)\left(\tfrac{1}{36}\right) + (100)\left(\tfrac{14}{36}\right) + (300)\left(\tfrac{1}{36}\right) + (-100)\left(\tfrac{20}{36}\right) = \boxed{-\$2.78}$$

Standard Deviation

$$\sigma = \sqrt{\sum x^2 \, P(X=x) - \mu^2} = \sqrt{(200)^2\left(\tfrac{1}{36}\right) + (100)^2\left(\tfrac{14}{36}\right) + (300)^2\left(\tfrac{1}{36}\right) + (-100)^2\left(\tfrac{20}{36}\right) - (-2.77\ldots)^2}$$

$$= \boxed{\$114.23}$$

Variance

$$\sigma^2 = (114.22714)^2 = \boxed{13047.83951}$$

c) Explain in words the meaning of the expected value calculated in part (b)

If you bet $100 on the field many times, it's as if you lose about $2.78 per bet.

d) Is this a good game for you to play? Why or why not?

Bad game for you because the expected value is negative.

2. (2 points) Consider the experiment where you flip a single coin 3 times. Define a random variable on this experiment.

$$X = \begin{cases} 1 & \text{if all flips are heads} \quad (\text{i.e. HHH}) \\ 4 & \text{if all flips are tails} \quad (\text{i.e. TTT}) \\ -2 & \text{otherwise.} \end{cases}$$